QP CODE: 23507140

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Reg No : .....

Name : .....

INTEGRATED MSC DEGREE EXAMINATION, OCTOBER 2023

Third Semester

INTEGRATED MSC BASIC SCIENCE-PHYSICS

COMPLEMENTARY - IPH3CM04 - MATHEMATICS - III DIFFERENTIAL EQUATIONS AND VECTOR CALCULUS

2021 Admission Onwards

A7C1DFDA

Time: 3 Hours Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Solve the differential equation $\frac{dy}{dx} = 1 + y^2$.
2. Obtain the differential equation associated with the primitive $y = Ax^2 + Bx + C$.
3. Examine whether the differential equation $(e^y + 1)\cos x dx + e^y \sin x dy = 0$ is exact or not.
4. What is the equation of the tangent plane to the surface S given by $F(x, y, z)$ at a point $P(x, y, z)$?
5. Define order and degree of a partial differential equation. Give example.
6. Define limit of a vector valued function.
7. Define countinuity of a vector valued function
8. Define gradient vector of f(x,y) at a point $P_0(x_0,y_0)$.
9. Give the properties of line integral over a curve.
10. Define the area of the surface $f(x, y, z) = C$ over a closed bounded region R.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Solve $x\frac{dy}{dx} + y = xy^3$.

12. Solve the initial value problem $\sin y dx + (2y + x\cos y)dy = 0,\ y(0) = \pi$.

13. Solve $\frac{dx}{xz(z^2+xy)} = \frac{dy}{-yz(z^2+xy)} = \frac{dz}{x^4}$.

14. Solve $py + qx = xyz^2(x^2 - y^2)$.

15. Find an equation for the tangent to the circle $x^2 + y^2 = 4$ at the point (0,-2)

16. Illustrate the difference rule and product rule for the function f(x,y) = x-y and g(x,y) =3y

17. Show that $F = (e^x \cos y + yz)\mathbf{i} + (xz - e^x \sin y)\mathbf{j} + (xy + z)\mathbf{k}$ is conservative and find a potential function for it.

18. Find the net outward flux of the field $F = \frac{x\mathbf{i}+y\mathbf{j}+z\mathbf{k}}{\rho^3}$, $\rho = \sqrt{x^2 + y^2 + z^2}$ across the boundary of the region $D : 0 < a^2 \leq x^2 + y^2 + z^2 \leq b^2$.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. a)Find the integrating factor and hence solve the initial value problem $2\sin y^2 dx + xy\cos y^2 dy = 0$, $y(2) = \sqrt{\frac{\pi}{2}}$.

    b) Solve $(xy^3 + y)dx + 2(x^2y^2 + x + y^4)dy = 0$.

20. Form a partial differential equation by eliminating the arbitrary function ϕ from $lx + my + nz = \phi(x^2 + y^2 + z^2)$.

21. (a) Show that if f is a twice differentiable function of x, then the curvature of the graph of y = f(x) is $\kappa = \frac{|f''(x)|}{[1+[f'(x)]^2]^{\frac{3}{2}}}$

    (b) Use the formula for κ to find the curvature of y = ln(cosx0, $(\frac{\pi}{2} < t < \frac{\pi}{2})$

22. Find the circulation of $F = 2x\mathbf{i} + 2z\mathbf{j} + 2y\mathbf{k}$ along the closed path consisting of the following three curves traversed in the direction of increasing t.

    $C_1 : r(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + t\mathbf{k}; 0 \leq t \leq \frac{\pi}{2}$

    $C_2 : r(t) = \mathbf{j} + \frac{\pi}{2}(1-t)\mathbf{k}; 0 \leq t \leq 1$

    $C_3 = r(t) = t\mathbf{i} + (1-t)\mathbf{j}; 0 \leq t \leq 1.$

(2×5=10 weightage)